The Seasonality of Momentum: Analysis of Tradability*

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Abstract

This paper shows that momentum-based strategies have exhibited high excess returns during the last several decades, especially in December and January. The effect of trading on prices, however, limits the amount that can be invested in such strategies. We find that after taking into account the price impact induced by trades, no more than $200 million can be invested before the apparent profit opportunities vanish. Thus, despite the failure of factor models to explain the persistence of momentum returns at the turn-of-the-year, actual trading does not award abnormal profits. Thus, the existence of momentum seasonality does not contradict the efficient market hypothesis.

JEL classification: G11; G14

Keywords: Momentum strategies; Transaction costs; Price impact; Market efficiency; January effect

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1. Introduction

During the past decade there has been growing literature on the predictability of stock returns based on the information contained in past returns. At very short horizons, such as a week or a month, returns are shown to have negative serial correlation (reversal), while at three to twelve months, they exhibit positive serial correlation (momentum). During longer horizons, such as three to five years, stock returns exhibit the reversal effect.[1] The momentum of individual stocks is extensively examined by Jegadeesh and Titman (1993). They document the out-performance of trading strategies based on past performance. They show that one can obtain superior returns simply by holding a zero-cost portfolio that consists of long positions in stocks that have out-performed in the past (*winners*), and short positions in stocks that have under-performed during the same period (*losers*). Others have shown that most of the returns to a momentum trading strategy are due to losers rather than to winners (see, e.g., Hong, Lim, and Stein, 2000; Grinblatt and Moskowitz, 2000). The momentum anomaly is further deepened by the existence of seasonal patterns. As discussed by Jegadeesh and Titman (1993) and Grinblatt and Moskowitz (2000), the usual momentum return continues throughout February-November, increases in December, but changes to a strong reversal in January. The explanation of the momentum anomaly has been the focus of many researches during the last several years. Until now, no measures of risk has been found that completely explain momentum returns. Most recently, Grundy and Martin (2001) study the risk sources of momentum strategies and conclude that while factor models can explain most of the variability of momentum returns, they fail to explain their mean returns (see, e.g., Jegadeesh and Titman, 2000).[2] Momentum has also been shown to be robust to international financial markets (see, e.g., Bhojraj and Swaminathan (2001)). Some view this unexplained persistence of momentum returns throughout the last several decades as one of the most serious challenges to the market efficiency hypothesis.

The purpose of this paper is to demonstrate that the market efficiency hypothesis is not violated by the above findings. This paper introduces new methods to assess whether trading strategies designed to exploit the observed seasonality remain profitable after considering transaction costs, such as trading fees and price concessions. This paper shows that the apparent excess returns of momentum strategies disappear when the price impact induced

[1] For evidence on short horizon reversal, see Poterba and Summers (1988), and Jegadeesh (1990); for momentum and long run reversal, see DeBondt and Thaler (1985), Jegadeesh and Titman (1993), and Grinblatt and Moskowitz (2000).

[2] Others, such as Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Hong and Stein (1999), explain the momentum continuation by forming behavioral models based on underreaction/delayed overreaction of investors to information.

by trades is considered. Our result stems from the fact that most of the demonstrated excess returns originate from small illiquid stocks that have extremely underperformed in the past.[3] The paper shows that an investor attempting to implement the momentum strategies documented in the literature will induce price pressures that will eliminate the excess returns.

To achieve our goal, we first construct momentum trading strategies, which are consistent with the existing literature. We focus only on momentum seasonality, i.e. December and January alone, because momentum strategies display the largest excess returns (in absolute values) during these months. Then, in light of the well documented small-firm effect during January[4], we examine the relation between momentum and size during turn-of-the-year. We find that small firms that have lost during the previous year bounce back in January more than other small firms (small losers earn 17.2% in January, while other small firms earn 8.4%). In contrast, large firms underperform in January independent of their past performance.[5] Based on this analysis, additional momentum/size strategies are generated, resulting in excess returns that are even higher than those analyzed in the literature.

The contribution of this paper is in testing whether the excess returns to momentum/size strategies during turn-of-the-year can be realized when price impacts of the strategies are considered. Price impacts of trades have recently received attention in microstructure literature (see Breen, Hodrick and Korajczyk (2000), and Huberman and Stanzl (2000) for admissible price impact functions, and Bertsimas and Lo (1998), and Almgren and Chriss (2001) for optimal execution of trades; See also Glosten and Harris (1988), and Hasbrouck (1991)). The importance of price pressure is also demonstrated by Knez and Ready (1996), who show empirically that transaction costs increase substantially as the size of an order approaches the quoted depth. Another example is Keim and Madhavan (1996), use data of

[3] Since we focus on turn-of-the-year investment strategies, it is important to mention the study of Keim (1989), who investigates whether one can exploit the high returns of small firms during January. He concludes that in light of large bid-ask spreads of small firms, an attempt to buy small firms at the ask in December and to sell them at the bid during January would not induce much profit. We expand on this observation and argue that even if one ignores bid-ask spreads, the large price concessions induced by the trades will eliminate profitability.

[4] The January effect has been extensively studied since the early 1980s. Banz (1981) discovered that, on average, firms with small-market capitalization outperform those with large-market capitalization. By adding a time dimension, Keim (1983) documents that roughly half of the annual size effect may be attributed to the returns during January. Explanation for the January effect include tax-loss selling (see, e.g., Constantinides, 1984; Dammon and Spatt, 1996; Dyl, 1977; Givoly and Ovadia, 1983; Lakonishok and Smidt, 1986; Reinganum, 1983; Chan, 1986) and window dressing by portfolio managers toward year-end (see, e.g., Haugen and Lakonishok, 1988; Ritter, 1988; Dyl and Maberly, 1992; Sias and Starks, 1997; Musto, 1997).

[5] These results are similar to those documented by Sias and Starks (1997) and Grinblatt and Moskowitz (2000).

large-block trades in the upstairs market to show that transaction costs increase with the size of the trade.[6]

Our analysis utilizes a linear measure for price impact of trades, one which is suitable for fast trading to exploit returns during the turn-of-the-year. Specifically, we employ the price-impact measure introduced by Breen, Hodrick, and Korajczyk (2000). This measure assumes that the impact on a stock price is proportionate to the net turnover induced by trading it.[7] The trading strategies are analyzed in two scenarios—as stand-alone strategies and as part of a larger managed portfolio. For each scenario we develop methods to quantify the dollar amount that could be profitably invested in momentum strategies, after taking into account price concessions induced by trades. In particular, we find the amount that can be invested so as to provide performance superior to that of different benchmarks. We find that even if one ignores the direct transaction costs, after taking into account the price impact of trades, no more than $200 million can be employed as a stand-alone strategy, before the apparent profit opportunities vanish. When a year-end strategy is considered as part of a portfolio that tracks a market benchmark, no more than $100 million can be invested in the strategy. Therefore, our conclusions support the market efficiency hypothesis to the extent that excess returns displayed by strategies that deploy large investment amounts cannot be achieved by naively following momentum/size-based strategies at turn-of-the-year.

The remainder of this paper is organized as follows. Section 2 describes the data. The formation of momentum trading strategies is explained in Section 3. Section 4 presents the methodology used to estimate transaction costs. Performance evaluation of different trading strategies is conducted in Section 5, followed by concluding remarks in Section 6.

2. Data

The data used for this research consist of all stocks included in the Center for Research in Security Prices (CRSP) monthly data files from December 1963 to December 1999. From 1964 to 1972, the CRSP data files include NYSE and AMEX stocks only; after 1972, NASDAQ stocks are added to the sample. In addition, we use intraday data from the TAQ database to measure price impacts for the analysis of the profits to momentum trading strategies after considering transaction costs. Since the TAQ database started recording transaction data only in 1993, the sample used for estimating price impacts is confined to

[6] Price impacts are also known as invisible costs (see Treynor (1994)).

[7] Mitchell and Pulvino (2001) utilize a similar measure of price impact to analyze the risk and returns to risk arbitrage.

the period 1993-1999.

3. Formation of Trading Strategies

Jegadeesh and Titman (1993) form different zero-cost trading strategies using various ranking and holding periods, such as one, three, six and twelve months. From our analysis we conclude that the most prominent strategy is the one known as the 12/1 equally weighted (EW) trading strategy.[8] This strategy ranks all stocks according to their past 12-month return every period.[9] The 12/1 strategy assumes a long position in the highest performing decile, whose stocks are referred to as winners, and a short position in the lowest performing decile, whose stocks are referred to as losers. The momentum deciles are formed using NYSE breakpoints.[10] These positions are held for one month, and the entire process is repeated every month. Fig. 1 plots the time-series averages of monthly returns of the 12/1 momentum strategy for every month in the year. Clearly, the highest momentum returns are exhibited during December, followed by a strong reversal during January. Focusing the discussion on the turn-of-year seasonality, Table 1 reports the returns of the 12/1 trading strategy separately for January, February-November and December, divided into premiums on losers and winners.[11] Two phenomena are observed. First, there is a clear seasonal pattern to the excess return of winners over losers. Throughout February-November, a momentum return of 1.6% per month is observed, followed by an increased return of 4.9% during December, and a very strong reversal of negative 6.1% in January. Since December and January exhibit the highest momentum returns (in absolute value), we focus our discussion throughout the paper on these months. Second, the winners minus losers returns during December and January stem mostly from losers. In addition, the January reversal has an asymmetric effect on winners and losers; the reversal occurs to losers while returns to winners are not significant. These results have been discussed in part by Jegadeesh and Titman (2000) and by Grinblatt and Moskowitz (2000). It might be argued that investors have digested this anomaly over

[8] Out of all the different trading strategies examined by Jegadeesh and Titman (1993), the 12/3 EW trading strategy is found to have the best performance. The return to this strategy is equivalent to the average return of three time-consecutive 12/1 strategies. Therefore, this paper focuses on the 12/1 trading strategy. This conclusion is also mentioned in Grinblatt and Moskowitz (2000).

[9] The last month's return is excluded due to microstructure effects, such as bid-ask bounce, as discussed in Roll (1984), Jegadeesh (1990), and Lo and MacKinlay (1990).

[10] Momentum strategies constructed without the use of NYSE breakpoints yield similar results. Later, we form size groups based on NYSE breakpoints, and therefore momentum groups are formed similarly.

[11] The figures reported are calculated as follows. First, cross-sectional averages are obtained for losers and winners every period. Then the time series average is reported. This procedure is similar to the Fama and MacBeth (1973) regression methodology.

time and that it is, therefore, a diminishing phenomenon. Nevertheless, a time series of the momentum returns, which is omitted here for the sake of brevity, reveals not only that the effect is not declining but actually has become even stronger over the last decade.[12]

As noted above, December and January exhibit the highest momentum returns (in absolute value), and therefore we focus our discussion throughout the paper on these months. In light of the well documented small-firm effect during January, the relation of size and momentum is first investigated. We form 20 size groups (independent of the momentum deciles), where the size of a firm is measured by its market capitalization, and the groups are constructed using NYSE breakpoints. We calculate the frequency of each momentum decile for every size group (we sort first by size and then assign momentum categories). The results are reported qualitatively in Fig. 2. It appears that about 30% of the small firms have been in the lowest momentum decile. In contrast, the distribution of past momentum for the large firms is fairly uniform. Note that Fig. 2 sheds light on the problem of endogeneity when analyzing the interaction between momentum and size. One might argue that small firms are firms that have extremely underperformed in the past, and that size and momentum, therefore, are highly correlated. Although we find evidence of some endogeneity, most small firms are not losers. In fact, more than 10% of the small firms are winners. Therefore, momentum and size are not nested and further analysis follows.

To analyze the interaction between momentum and size, a set of double sorts is performed, as reported in Table 2. These sorts differ from the previous sorts in that previously, size and momentum groups were sorted independently of each other. In contrast, here all stocks are first sorted into five quintiles of size, and then the firms within each quintile are sorted into five momentum groups.[13] This way one may analyze the distribution of momentum conditional on size.[14] The results indicate that returns seem to increase with momentum in December, for all size group. In January, however, the behavior of the conditional size distribution differs. Small firms exhibit a strong momentum reversal; low momentum small firms earn a high excess return of 8%, while the excess returns to high momentum small firms are nonsignificant. In contrast, large firms observe a negative excess return of about 5%, independent of their past performance.[15]

[12] Momentum returns have become more significant to both winners and losers, throught the entire year.

[13] As our previous analysis, all sorts are conducted using NYSE breakpoints.

[14] Sorting first by momentum groups and then by characteristics result with very similar group returns, and therefore are excluded from the paper.

[15] Our main concern in this paper is the tradability of momentum strategies. Therefore, we have also performed double sorts of momentum and each of the following measures: turnover, abnormal volume, and percentage of institutional ownership. Since all of these measures are highly correlated with market capitalization, the results of these analyses are very similar to that of momentum and size, and are, therefore,

The trading strategies examined so far utilized all stocks traded in NYSE, AMEX, and NASDAQ. However, since small firms account for much of the high premiums—especially during January—one may already at this point question the feasibility of the proposed strategies on the basis of the relatively high illiquidity of small firms. Before involving an explicit measure of transaction costs, we perform some tests for the robust quality of the results. First, in order to loosely account for the liquidity of large firms and the lack of it in small firms, one might suggest the formation of a market value-weighted portfolio, instead of the equally weighted portfolio analyzed so far throughout this paper. However, by constructing such portfolios, the effect of small firms may be masked by the returns of the largest firms. Consequently, the 12/1 value-weighted (VW) trading strategy, which emphasizes the returns of largest firms, is expected to earn less profits.[16] To illustrate this intuition, we recalculate the results using market values as weights in Table 1, Panel B. The results clearly support our prediction: the seasonality of momentum is no longer observed. We also replicate the size/momentum sorts for value-weighted returns, reported in Panel B of Table 2. We find similar results to those documented in Table 2, Panel A.[17]

To summarize the finding in this section, we have established the predictability of returns during turn-of-the-year based on momentum and size. In what follows we use these results to construct additional trading strategies that seem to out-perform the year-end trading strategies documented in the literature. Our analysis shows, however, that even though these strategies display higher excess returns, they are much less tractable once price impacts of trades are considered.

4. Estimation of Transaction Costs

The trading strategies studied thus far in this paper do not consider various transaction costs. This is disturbing in light of the crucial role of the smallest firms in the profitability of the trading strategies, which are often relatively costly to trade. In this section we test whether momentum trading strategies remain profitable after incorporating transaction costs. The transaction costs consist of two parts—the costs associated with microstructure

excluded from this section.

[16] Note that the magnitude of market capitalization of the large firms is of several orders higher than that of the small firms. In fact, the firms in the largest size group are so huge compared to the rest, that even the mid-cap firms would not have a fair representation in a value-weighted portfolio.

[17] We conduct additional tests of robustness by replicating the results for different time periods, such as December 1963–January 1981 and February 1981–December 1999, and different subsets of the sample, such as using stocks from different exchanges alone (NYSE and NASDAQ), and using only firms with a market capitalization above $50 million. We find similar patterns in all data sets.

effects[18], such as price impacts while attempting to buy/sell stocks (stocks' illiquidity), and the direct trading costs in the form of brokerage fees.

First, we define the following variables, with the subscript t indicating that the variables are measured at time t: p represents the portfolio analyzed; MVE_t^i is the market value of equity of firm i; w_t is the normalized weight vector that records the weight w_t^i of each stock i in portfolio p; R_t^i is the raw return—before transaction costs—of stock i; R_t^p is the weighted raw return of portfolio p; and x_t is the dollar amount invested in the portfolio. The price impact is estimated using the measure developed in Breen, Hodrick, and Korajczyk (2000). This measure assumes a linear relation of relative price movements and net turnover, and is calculated through the following regression model:

$$\frac{\Delta p_t^i}{p_t^i} = \beta^i \times Turnover_t^i + \varepsilon_t^i \tag{1}$$

where $p_{i,t}$ is the price of asset i at time t prior to the transaction, $\Delta p_{i,t}$ is the price impact associated with the transaction, $\beta_{i,t}$ is asset i's price impact coefficient, and $Turnover_{i,t}$ is the net (signed) number shares traded divided by the number of shares outstanding for firm i. Purchases correspond to positive values of $Turnover_{i,t}$ and sales correspond to negative values.[19] This specification is motivated by the linear pricing rule of Kyle (1985), which expresses absolute price changes as a linear function of net volume. Breen, Hodrick, and Korajczyk (2000) motivate the use of scaled measures as a means of obtaining more meaningful cross-sectional and time series comparisons of price impact. Using net turnover rather than net volume preserves linearity since shares outstanding are essentially fixed over the observation interval used to estimate the price impact. Using returns rather than price changes does induce non-linearity in the price impact. Table 3 reports the averages and standard deviations of the time series of β medians[20] of different stock groups, separately for December and January. In general, small firms have a higher price impact than large firms[21], and similarly, losers have a higher price impact than winners. However, a deeper look into size and momentum reveals that small losers have much higher price impacts than small winners. In contrast, large losers have a slightly lower price impact than large winners.[22]

[18] For excellent surveys of the microstructure literature see Madhavan (2000), and O'Hara (1995).

[19] In order to determine whether a trade is a buy or a sell we compare the trade price to the mid-point of the bid and ask prices. Following Lee and Ready (1991), we use the bid and ask quotes in the TAQ database as of five seconds prior to the trade. After determinig the nature of the trades, we calculate the net turnover for each five-minute trading interval. Returns are calculated using percentage change in the last traded price.

[20] Due to the existence of several outliers in the cross-sectional distribution of β, medians rather than averages are reported.

[21] Loeb (1983) and Keim and Madhavan (1996) study the price impact of large-block trades. They find that price impact decreases with firm size, which is consistent with our findings in Table 3.

[22] The literature includes other estimation methods of price impacts. For example, Knez and Ready

The direct trading costs, TC_t, are estimated as a fixed amount per share traded; until 1979: \$0.10, 1980-1989: \$0.05, and after 1990: \$0.04.[23] We assume that the positions entered in the beginning of each month are fully liquidated at the end of that month, i.e., the transaction costs are induced twice every investment period.

The net return to a portfolio after considering transaction costs, r_t^p, is calculated as $r_t^p = R_t^p - PI_t^p - TC_t^p$, where PI_t^p denotes the price impact (in terms of return) induced by trading, and TC_t^p denotes the direct trading costs (per dollar invested). Denote n_t^i as the number of shares of stock i that are traded at time t. Therefore, n_t^i is calculated through

$$n_t^i = w_t^i \times \frac{x_t}{p_t^i} \tag{2}$$

Using the above and the relation $MVE_t^i = p_t^i \times \mathit{Shares\ Outstanding}_t^i$, we calculate PI_t^p and TC_t^p as follows

$$\begin{aligned}
PI_t^p &= [\mathit{Costs\ of\ entering\ position}] + [\mathit{Costs\ of\ closing\ position}] \\
&= \sum_{i \in p} w_t^i \frac{n_t^i}{\mathit{Shares\ Outstanding}_t^i} \beta_t^i + \sum_{i \in p} w_t^i \left[\frac{p_{t+1}^i}{p_t^i}\right] \frac{n_t^i}{\mathit{Shares\ Outstanding}_t^i} \beta_t^i \\
&= x_t \sum_{i \in p} \frac{\left[w_t^i\right]^2 \beta_t^i}{MVE_t^i} \left[1 + \frac{p_{t+1}^i}{p_t^i}\right] && (3) \\
TC_t^p &= \frac{2TC_t}{x_t} \sum_{i \in p} n_t^i = 2TC_t \sum_{i \in p} \frac{w_t^i}{p_t^i} && (4)
\end{aligned}$$

Several comments are appropriate. First, notice that p_{t+1}^i / p_t^i does not measure the raw return of stock i, since the latter includes dividend payments during the trading period. Second, the only variable that is proportionate to x_t is the price impact. Therefore, we normalize the price impact $\overline{PI}_t^p = PI_t^p / x_t$ and rewrite r_t^p as

$$r_t^p = R_t^p - x_t \overline{PI}_t^p - TC_t^p \tag{5}$$

Third, to conduct a conservative analysis we assume that the execution of a trade is divided into several smaller trades. Specifically, our analysis below assumes that ten equal trades are necessary to execute an entire trade of any given stock, consequently reducing the price impact PI_t^p by a factor of ten. Last, the above formulation of net returns clearly illustrates

(1996) use nonparametric regression techniques to estimate price improvements. They study the weekly predictability of returns to small firms and show that transaction costs would turn the attempt to exploit this phenomena unprofitable.

[23] Prior to 1975, trading fees were regulated by the NYSE; after 1975, brokerage houses were free to charge competitively. The trading costs stated above are an approximation.

two types of transaction costs. The direct trading costs, TC_t^p, enter as a percentage of stock price, i.e., a fixed cost in terms of return. This type of transaction cost is most commonly used in the asset pricing literature, nevertheless, it does not vary with the amount of the investment. In contrast, the price-impact measure, $\overline{PI}_t^p$, is a return that is proportionate to the size of the trade order. With this formulation of transaction costs we proceed in the next section to evaluate the performance of different trading strategies that attempt to capture the turn-of-the-year excess returns.

5. Performance Evaluation of Strategies

In the previous section we developed a general framework that includes trading costs. Clearly, the higher the position taken in a trading strategy, the higher the trading costs. We now proceed to assess the performance of trading strategies using measures of mean return and Sharpe ratio. The evaluation of these strategies depend on the context in which they are implemented. Two scenarios are considered: stand-alone strategies and strategies as part of a larger managed portfolio. In the first case, we solve for the maximum dollar amount that would make the performance of the evaluated trading strategy break even with the performance of different benchmarks. When a trading strategy is examined as part of a larger managed portfolio, the optimal dollar amount to be allocated to the trading strategy is first computed. We assume a managed portfolio that tracks the performance of a benchmarks. Thus, in this context, an optimal allocation is defined as the amount to be invested in a trading strategy so as to provide with the highest Sharpe ratio of a portfolio combined out of the trading strategy and the benchmark. Then, we calculate the marginal contribution of the trading strategy to the overall performance of the portfolio. The benchmarks used for the analysis are the following indexes provided by CRSP: value-weighted index (VW market) and equally weighted index (EW market) of all NYSE, AMEX, and NASDAQ stocks, and Standard and Poor's 500 index (S&P). Notice that the performance of VW market and S&P is biased toward that of large firms, while the performance of EW market is biased toward the performance of small firms. In addition, in light of our finding regarding the role of size in the turn-of-year effect, we add two more informative benchmarks—the small size quintile index and the large size quintile index. The performance measures of the different benchmarks during December and January are reported in Table 4. In what follows, these measures are compared with the performance of the trading strategies to motivate the implementation of these trading strategies.

5.1. Stand-Alone Momentum Strategies

In order to estimate the performance of different trading strategies, we first develop the general formulas of mean and Sharpe ratio to include transaction costs. In order to account for different values of cash over the years, we use the total market capitalization of NYSE, AMEX, and NASDAQ stocks as a deflator index. Denote i_t as this deflator. We compare the trading strategies according to a constant real investment x (measured in dollars valued at the end of 1999), such that $x_t = i_t x$. With the assumption of a constant investment position, the net return of a portfolio is given by

$$r_t^p = R_t^p - x\overline{PI}_t^p i_t - TC_t^p \tag{6}$$

In what follows, it is useful to define the following variables (r_t^f denotes the risk-free return at time t)

$$\begin{array}{ll} a_t = R_t^p - TC_t^p - r_t^f & b_t = \overline{PI}_t^p i_t \\ a = \frac{1}{T}\sum\limits_{t=1}^{T} a_t & b = \frac{1}{T}\sum\limits_{t=1}^{T} b_t \\ \overline{a_t} = a_t - a & \overline{b_t} = b_t - b \end{array} \tag{7}$$

Denote r^p and r^b as the estimates of the unconditional mean returns of the trading strategy and of the benchmark, respectively.

$$\begin{aligned} r^p &= \frac{1}{T}\sum_{t=1}^{T} r_t^p \\ &= a + \frac{1}{T}\sum_{t=1}^{T} r_t^f - xb \end{aligned} \tag{8}$$

The maximum allowable position, x^μ, that satisfies $r^p \geq r^b$, is given by

$$x^\mu = \frac{a + \frac{1}{T}\sum\limits_{t=1}^{T} r_t^f - r^b}{b} \tag{9}$$

Denote SR^p and SR^b as the Sharpe ratios of the trading strategy and of the benchmark, respectively. Under the notation above, SR^p translates to

$$SR^p = \frac{a - xb}{\sqrt{\frac{1}{T-1}\sum\limits_{t=1}^{T}\left[\overline{a_t} - x\overline{b_t}\right]^2}} \tag{10}$$

Our goal is to determine the maximum allowable position, x^{SR}, that satisfies $SR^p \geq SR^b$. To solve for x^{SR} we must solve the following quadratic form, derive directly from (10)

$$\left[\frac{SR^b}{T-1}\sum_{t=1}^{T}\overline{b_t}^2 - b^2\right]\left(x^{SR}\right)^2 - 2\left[\frac{SR^b}{T-1}\sum_{t=1}^{T}\overline{a_t}\overline{b_t} - ab\right]x^{SR} + \left[\frac{SR^b}{T-1}\sum_{t=1}^{T}\overline{a_t}^2 - a^2\right] = 0 \qquad (11)$$

An upper bound for a position that induces a positive Sharpe ratio is denoted as x^{ub} and calculated directly using the second inequality, $x^{ub} = a/b$. This is the position that breaks even the return of the trading strategy with the risk-free return. Since there are two solutions to the quadratic formulation given above, x^{SR} is chosen as the solution with the higher value as long as it is less or equal to x^{ub}.[24]

In order to implement the methodology described above, we must determine the chosen trading strategy. Explicitly, the group of stocks to be traded, the weight vector, buy or short sell position, and the holding period must all be stated. This paper examines the following trading strategies. First, we examine strategies based on the 12/1 momentum ranking. Since losers and winners behave differently during the turn-of-year, the two deciles are examined independently. Also, in light of the reversal during January, we examine the implementation of every strategy on a monthly basis, i.e., the position is taken in the beginning of the month and is closed at the end of that month. Also, we stress that the transaction-cost model used here explicitly assumes a very short period for the execution of trade orders, so that one could exploit the high monthly raw returns.[25] In light of the results mentioned in Section 3 regarding the interaction of size and momentum, we also examine strategies based on the different combinations of small/large and losers/winners.[26] As a preliminary step, we calculate in Table 5 the maximum attainable means, Sharpe ratios, and appraisal ratios, i.e.,

[24]The empirical estimates of a, $\overline{a_t}$, b, $\overline{b_t}$, and SR^b in our sample always result with choosing the smaller solution of the quadratic form problem.

[25]Notice, trading slowly would induce lower price impacts, but would not be appropriately used with monthly returns. Since our goal is to examine whether an investor can induce profits by following the strategies documented in the literature, we confine the analysis for monthly trading, and thus focus on monthly returns alone.

[26]In order to have a more accurate description of turn-of-year phenomena, we make several adjustments to the measures of past returns and the formation of size groups. First, we have previously measured momentum returns using the 12/1 ranking. However, due to the unusual returns observed during December and January, we shorten our ranking period and use 10/1, so that in the beginning of December, we rank the stocks according to their cumulative returns during February-October. Second, the categorization of firms into size groups was originally conducted in the beginning of every month, allowing firms to switch size groups according to their relative value of market capitalization. Now, however, we rank the stocks only once—at the beginning of December—and hold the size groups fixed during January. Last, in light of the endogeneity of size and momentum that has been established previously (see Fig. 2), the size groups are formed according to their relative market capitalization in the beginning of February. This way the measure of size and momentum are orthogonal, i.e., the size groups are determined prior to the performance of the stocks during February-October.

assuming no price impact. This way the effect of the trading fees is distinguished from that of the price impacts. We note that trading fees notably reduce the performance measures. Also notice that the decision to assume a short or long position is determined by the sign of mean return without fees. A mean return higher (lower) than the risk-free rate suggests implementing a long (short) position. Table 5 also reveals the underlying motivation for year-end trading: small losers earn 17.2% (EW) and 12% (VW), with a Sharpe ratio of 1.14 (EW) and 0.90 (VW), while Table 4 shows that the market index earns 6.7% (EW) and 2.4% (VW), with a Sharpe ratio of 0.88 (EW) and 0.36 (VW).

After determining the strategies, we proceed to calculate maximum allowable positions that would break even the performance measures of the trading strategies with those of the different benchmarks. Tables 6-8 report these positions for equally weighted and value-weighted strategies during December and January. One should be cautious during the analysis of these stated numbers, as the relative values are perhaps more trustworthy than the absolute values. Therefore, we mostly compare the figures of different strategies, months, and performance measures. We begin with the analysis of the 12/1 trading strategy in Table 6. In general, comparing performance measures with and without fees, it is clear that fees wipe out most of the investment positions. However, since large investment institutions are more interested in their price impacts rather than transaction fees, we proceed with the discussion of the results without fees. Also, EW losers-based strategies as well as VW losers-based strategies are not exploitable during December. Comparing means to Sharpe ratios of winners during December (EW and VW) reveals that the maximum attainable positions to break even with large firms biased indices drop roughly by 30%-40%, while break-even figures with small firms-biased indices drop very little. During January mostly large biased indices are achievable (EW/VW, losers/winners). However, a closer look at EW strategy reveals that, contrary to 'winners' during December, 'losers' during January experience the largest percentage drop from means to Sharpe ratios for the break-even figures with the small firms biased indices rather than large firms biased indices. However, for the respective VW strategies, losers during January exhibit a substantial drop from means to Sharpe ratios. The most striking observation is the large figures obtained by the winners-based VW strategy during December in general, and relative to the respective EW strategy in particular (as much as ten times higher). The latter nicely illustrates the reason it is much better to use value-weighted measures of returns—rather than equally weighted measures—as a proxy of tradability of strategies.

The analysis of strategies that utilize only small firms is provided in Table 7. In general, small firm-based strategies follow the same patterns displayed by the 12/1 momentum-based strategies discussed above. However, all break-even levels are significantly smaller than those of the respective 12/1 strategies. The difference is approximately five times less, excluding VW winners during December, which drop about 90%. Consequently, EW and VW break-even figures are much closer than those in Table 6. The general drop in break-even levels is explained by two effects that influence the tradability of small firms. First, as already shown in Table 3, the price impact measures β_i of small firms are higher than those of firms in general. Second, using only small firms reduces the set of stocks available for trade, thereby leaving a larger amount of the total investment position invested in each stock.[27] Consequently, *net turnovers* of small firms are higher than those exhibited in the 12/1 strategies. The increase in both components, β and *net turnover*, increases the average price impact that small firm-based strategies possess relative to all stocks-based strategies, such as the 12/1 momentum strategy. The drop during December of VW winners implies that large firms induce the majority of the effect during December. EW and VW figures are more similar since the distribution of firm size is less skewed at the left tail. Finally, although we initially found small losers during January to be the most attractive (according to both mean and Sharpe ratio measures), it turns out that in the presence of transaction costs, a strategy designed to exploit this phenomena is less likely to succeed.[28] In general, large firm-based strategies are ineffective except that they may beat the S&P in mean measures.

To conclude, trading strategies designed to exploit the January effect are less attractive after considering transaction costs. It seems that VW 12/1 winners strategy during December is the most tractable of all turn-of-the-year strategies we have considered.

5.2. Momentum Strategies as Part of a Managed Portfolio

The question of implementing a trading strategy as part of a managed portfolio translates to finding whether such a strategy enhances the overall Sharpe ratio that may be obtained. In a mean and standard deviation setting, this problem is also equivalent to that of obtaining the new tangency portfolio in the presence of the proposed year-end strategy. We start by assuming a dollar amount y that is invested in a benchmark portfolio. Then, we introduce the year-end trading strategy and calculate the constant optimal fraction ω to be invested

[27] As discussed above, size quintiles are formed using NYSE breakpoints. Consequently, there are many more firms in the small size quintile than in any other size quintile. So, in fact the use of NYSE breakpoints makes the small size strategy more tradable than if quintiles of equal number of shares were used.

[28] Keim (1989) investigates whether one can exploit the high returns of small firms during January. He finds that most of the trade prices of small firms during December are quoted at the bid, as volume of small firms is generally biased towards seller-initiated. In January, trade prices are generally quoted at the ask. In light of large bid-ask spreads of small firms, an attempt to buy small firms at the ask in December and to sell them at the bid during January would not induce much profit. This observation strengthens our conclusion.

in this strategy, such that the maximum Sharpe ratio of the total investment is obtained. Relating this to x (discussed in the previous subsection), we have $y = \omega x$. In this section we focus only on the asset allocation problem in the presence of price concessions in absence of trading fees[29]. The Sharpe ratio of the total portfolio is given by

$$\begin{aligned} SR^p(y) &= \frac{\omega r^P + (1-\omega) r^b - r^f}{\sqrt{Var\left[\omega r_t^p + (1-\omega) r_t^b - r_t^f\right]}} \\ &= \frac{\omega(R^p - r^b) - \omega^2 y \overline{PI}_t^p i_t + r_t^b - r_t^f}{\sqrt{Var\left[\omega(R_t^p - r_t^b) - \omega^2 y \overline{PI}_t^p i_t + r_t^b - r_t^f\right]}} \end{aligned} \tag{12}$$

In what follows, it is useful to define the following variables

$$\begin{array}{lll} a_t = R_t^p - r_t^b & b_t = \overline{PI}_t^p i_t & c_t = r_t^b - r_t^f \\ a = \frac{1}{T}\sum_{t=1}^{T} a_t & b = \frac{1}{T}\sum_{t=1}^{T} b_t & c = \frac{1}{T}\sum_{t=1}^{T} c_t \\ \overline{a_t} = a_t - a & \overline{b_t} = b_t - b & \overline{c_t} = c_t - c \end{array} \tag{13}$$

The estimated Sharpe ratio of the managed portfolio is calculated as

$$SR^p(y) = \frac{a\omega - by\omega^2 + c}{\sqrt{\frac{1}{T-1}\sum_{t=1}^{T}\left[\overline{a_t}\omega - \overline{b_t} y\omega^2 + \overline{c_t}\right]^2}} \tag{14}$$

For any given portfolio size y, we calculate the optimal weight of the trading strategy, such that it maximizes $SR^p(y)$.[30] Then it is possible to calculate the marginal return added to the total portfolio by including the trading strategy. This marginal return is given by $a\omega - by\omega^2$, which is the difference between the return on the portfolio with the trading strategy and the return on the benchmark portfolio.

Similar to the methodology described in the previous subsection, we employ the analysis above to different year-end trading strategies and different benchmarks. However, here the question of whether to engage in a short/long position of a trading strategy is endogenously determined by the maximization of the total Sharpe ratio.[31] The results of the analysis

[29] Trading fees are excluded from our analysis because the costs induced by trading activities of large funds, in practice, are mostly due to price impacts rather than trading fees.

[30] Solving for the first order conditions for optimality involve the solution of a fourth order equation. We note that in our sample this equation always consists of two real roots (for any given combination of year-end trading strategy, benchmark, and total size of porfolio), one of which is feasible.

[31] Notice that the price impact enters as a negative return no matter the sign of ω, since the related term

are reported in Figs. 3-5. These figures illustrate, for example, that to achieve an optimal allocation of the EW losers trading strategy one would not invest more than $500 million in this strategy during January. Specifically for small losers, the investment should not exceed $250 million. More important, the marginal returns to the total portfolio are significantly reduced with fund size; a fund of $600 million in assets should invest about $100 million in an EW small losers strategy during January, however the marginal returns as a result would be very close to zero. Similar to the conclusions reached in Tables 6-8, value-weighted strategies are, in general, more attractive than equally weighted strategies, and large-based strategies are more attractive than small-based strategies.

6. Conclusions

The persistence of momentum returns throughout the last several decades is perceived as one of the most serious challenges to the market efficiency hypothesis. This paper attempts to test whether excess returns can actually be realized at the turn-of-the-year by following the momentum trading strategies documented in the literature. The analyses of these strategies in the literature do not consider transaction costs. We develop a methodology to incorporate transaction costs—such as trading fees and price impacts—and reexamine the performance of various momentum-based trading strategies. We show that trading strategies designed to capture the turn-of-year momentum/size behavior, especially those based on exloiting the behavior of losing/small firms, are unlikely to result in profit. Nevertheless, it is important to emphasize that these results do not indicate that momentum strategies in general are not profitable. The crucial input to a trading strategy is the holding period. In this paper, only short holding periods of one month—and a suitable transaction model to capture fast trading—were employed. Considering a longer investment horizon may prove to be profitable, as it would allow for slower trading and consequently reduce the importance of price concessions. However, since December and January exhibit the largest returns to momentum-based strategies, more research, both theoretical and empirical, is needed to estimate price-pressure functions for different trading scenarios. Once this is accomplished, the profitability of momentum trading strategies throughout the entire year can be further examined.

is a function of ω^2. This waives the predetermination of whether the optimal strategy is short selling or long position. Also, the performance of the total portfolio is influenced by the correlation of the trading strategy and the benchmark, so it is not obvious ex-ante whether to sell short or to go long. This is contrary to the analysis in the previous subsection, which used only the excess return on the risk-free asset to predetermine the optimal strategy.

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Table 1
Momentum trading strategies

We first sort all NYSE, AMEX, and NASDAQ stocks at the beginning of every month by their past 12-month return (excluding the last month). The bottom decile (losers) and top decile (winners) are then analyzed. Cross-sectional averages are computed every month to form time series of returns of losers and winners. The returns computed are excess of the CRSP market index. The time-series averages of returns as well as the associated *t*-statistics are reported below. (December and January are reported separately.) The analysis uses monthly return data for the period December 1963 to December 1999. All sorts by past performance employ NYSE breakpoints. Panel A uses the CRSP equally weighted index and equally weighted cross-sectional averages. Panel B uses the CRSP value-weighted index and computes the cross-sectional averages using market capitalization as weights.

Panel A: 12/1 equally weighted trading strategy				*Panel B: 12/1 value-weighted trading strategy*			
Period	Winners minus Losers	Losers	Winners	Period	Winners minus Losers	Losers	Winners
Feb-Nov	0.0156	-0.0077	0.0079	*Feb-Nov*	0.0179	-0.0107	0.0072
	8.40	-5.77	6.62		8.82	-5.57	5.27
Dec	0.0491	-0.0271	0.0220	*Dec*	0.0348	-0.0197	0.0150
	5.76	-5.49	4.07		3.56	-3.85	2.47
Jan	-0.0606	0.0504	-0.0102	*Jan*	-0.0112	0.0173	0.0061
	-4.57	5.55	-1.69		-0.84	2.07	0.89

Table 2
Double sorts of size/momentum

We first sort all NYSE, AMEX, and NASDAQ stocks into five quintiles at the beginning of every month according to their market capitalization. Then all stocks in every size quintile are sorted according to their past 12-month return (excluding the last month). Cross-sectional averages of monthly returns (excess of a market benchmark) are performed separately for December and January, every year, to form a time series of group returns. The time-series averages of returns and the associated t-statistics are reported below. In Panel A, equal weights are used to calculate the cross-sectional averages, and the CRSP equally weighed index is used as a benchmark. In Panel B, value weights are used to calculate the cross-sectional averages, and the CRSP value-weighed index is used as a benchmark. The analysis employs monthly return data for the period December 1963 to December 1999. All sorts by past performance and market capitalization are based on NYSE breakpoints.

Panel A: Equally weighted strategy

			December		
	low		Momentum		*high*
small	-0.0315	-0.0135	-0.0065	0.0023	0.0098
	-4.91	-3.59	-2.23	0.76	3.32
	-0.0107	0.0046	0.0071	0.0103	0.0224
	-3.97	1.43	2.43	3.67	5.16
Size	-0.0012	0.0047	0.0105	0.0154	0.0252
	-0.34	1.22	2.80	4.29	4.44
	-0.0024	0.0050	0.0075	0.0149	0.0220
	-0.59	1.04	1.67	3.12	3.28
large	0.0004	0.0031	0.0074	0.0095	0.0174
	0.07	0.57	1.41	1.63	2.17

			January		
	low		Momentum		*high*
small	0.0778	0.0306	0.0197	0.0099	0.0091
	7.01	5.84	4.06	3.04	1.83
	-0.0105	-0.0092	-0.0164	-0.0197	-0.0219
	-1.69	-2.44	-4.20	-5.28	-4.85
Size	-0.0244	-0.0245	-0.0291	-0.0333	-0.0336
	-5.53	-6.13	-8.32	-9.87	-6.36
	-0.0326	-0.0384	-0.0394	-0.0427	-0.0410
	-5.75	-7.80	-8.62	-8.71	-6.09
large	-0.0427	-0.0463	-0.0484	-0.0481	-0.0468
	-6.52	-7.25	-6.53	-6.02	-4.91

Panel B: Value-weighted strategy

			December		
	low		Momentum		*high*
small	-0.0373	-0.0184	-0.0098	-0.0007	0.0069
	-3.75	-2.45	-1.56	-0.10	1.18
	-0.0158	-0.0017	0.0003	0.0040	0.0154
	-2.89	-0.31	0.07	1.09	3.18
Size	-0.0067	-0.0027	0.0038	0.0089	0.0179
	-1.38	-0.61	1.13	2.79	3.84
	-0.0098	-0.0026	0.0006	0.0077	0.0142
	-2.31	-0.69	0.25	2.78	3.60
large	-0.0111	-0.0033	-0.0013	0.0006	0.0110
	-2.31	-0.74	-0.37	0.20	2.24

			January		
	low		Momentum		*high*
small	0.0823	0.0572	0.0528	0.0422	0.0411
	5.57	5.64	5.56	5.73	5.28
	0.0308	0.0334	0.0253	0.0228	0.0208
	2.93	4.07	3.88	3.80	3.05
Size	0.0177	0.0175	0.0132	0.0091	0.0099
	2.21	2.66	2.35	1.72	1.64
	0.0095	0.0057	0.0036	0.0001	0.0011
	1.32	1.10	0.79	0.02	0.23
large	-0.0007	-0.0057	-0.0051	-0.0044	-0.0044
	-0.13	-1.85	-1.80	-1.12	-0.73

Table 3
Price impacts of size and momentum groups

We estimate the price impact coefficients 🕓$_i$ through the following regression model: $PI_{i,t}$ = 🕓$_i$ $\times$ $Turnover_{i,t} + \varepsilon_{i,t}$, where $PI_{i,t}$ is the price impact (in the form of return) of stock i during a five-minute interval beginning at time t, and $Turnover_{i,t}$ is the net turnover during the same time interval. The regressions employ data pooled separately for December and January for every stock. After computing price-impact coefficients for every stock, time-series averages of portfolio medians are reported along with standard deviations. The size and momentum portfolios are defined as follows. In Panel A, all stocks are sorted at the beginning of each month according to their past 12-month return (excluding the last month). The top decile is defined as winners and the bottom decile as losers. Independent of momentum, all stocks are sorted at the beginning of every month according to their market capitalization (using NYSE breakpoints). The top quintile is defined as large firms and the bottom quintile as small firms. In Panel B, we first sort all stocks at the beginning of February according to their market capitalization. The bottom quintile (small firms) and the top quintile (large firms) are then held fixed throughout the year. The stocks in each size quintile are then sorted according to their cumulative returns during February-October. The bottom performance decile is denoted as losers, while the top decile is denoted as winners. This is done for small and large firms separately. These four size/momentum portfolios are held fixed from November until February of the following year. The analysis employs intraday data of NYSE, AMEX, and NASDAQ stocks from the TAQ database for all months of December and January between 1993 and 1999.

Panel A: Independent sorts of momentum and size

	December		January	
	Mean	Std	Mean	Std
Momentum				
Losers	9.98	1.22	10.03	1.91
Winners	6.70	1.00	6.78	1.05
Size				
Small	10.07	2.26	11.37	3.02
Large	8.59	1.53	8.51	0.58

Panel B: Conditional size/momentum sorts

	December		January	
	Mean	Std	Mean	Std
Small				
Losers	14.36	3.51	20.36	12.59
Winners	8.87	1.54	9.94	2.06
Large				
Losers	8.21	1.31	8.37	0.93
Winners	9.04	2.85	8.64	2.20

Table 4
Means, Sharpe ratios, and appraisal ratios of benchmarks

We calculate different performance measures of several benchmarks for December and January. Market VW and Market EW are the CRSP value-weighted index and equally weighted index, respectively, using all NYSE, AMEX, and NASDAQ stocks. Small cap index and large cap index are the bottom and top quintiles of market capitalization (using NYSE breakpoints, value-weighted). The mean return is the average return during these months. The Sharpe ratio is the average excess return of a benchmark over the risk-free rate divided by the standard deviation of this excess return. The appraisal ratio is the average excess return of a benchmark over the Market VW index divided by the standard deviation of this excess return. The analysis employs monthly data of the period December 1963 to December 1999.

	December			January		
	Mean	Sharpe	Appraisal	Mean	Sharpe	Appraisal
Benchmark						
Market VW	0.0195	0.40	-	0.0238	0.36	-
Market EW	0.0127	0.18	-0.23	0.0671	0.88	1.07
S&P 500	0.0166	0.33	-0.50	0.0120	0.28	-0.52
Small Cap Index	0.0159	0.18	-0.04	0.0727	0.80	0.92
Large Cap Index	0.0185	0.31	0.04	0.0189	0.23	-0.18
Risk free	0.0050	-	-	0.0048	-	-

Table 5

Maximum attainable performance measures of trading strategies

We calculate the maximum attainable performance measures (mean returns, Sharpe ratios, and appraisal ratios) of different trading strategies. These figures stem from having no price impacts. Negative figures are shown in brackets. The strategies investigated have one-month holding period. The definitions of losers and winners are different in each panel. In Panel A, all NYSE, AMEX, and NASDAQ stocks are sorted at the beginning of every month according to their past 12-month performance (excluding the last month). The bottom decile is denoted as losers while the top decile is denoted as winners. In Panels B and C, first all stocks are sorted according to their market capitalization at the beginning of February, using NYSE breakpoints. The bottom quintile is defined as small firms and the top quintile as large firms. The stocks in the small and large quintiles are then separately sorted according to their cumulative returns during February-October. The bottom performance decile in each size quintile is denoted as losers while the top decile is denoted as winners. Since losers—of all stocks and of small firms—earn less than the risk-free rate during December, the chosen strategy is a short position. All other strategies are long positions. Equally weighted and value-weighted strategies are both analyzed, and all strategies are analyzed with and without the existence of trading fees. The analysis employs monthly return data (December and January only) for the period December 1963 to December 1999.

Panel A: All firms

	Equally weighted strategies			
	December		January	
	Losers	Winners	Losers	Winners
Strategy	Short	Long	Long	Long
Mean (no fees)	0.0142	0.0347	0.1183	0.0579
Mean (with fees)	(0.0676)	0.0125	0.0316	0.0334
Sharpe (no fees)	0.30	0.51	0.98	0.74
Sharpe (with fees)	0.81	0.12	0.23	0.42
Appraisal (no fees)	(0.62)	0.42	1.08	0.78

	Value-weighted strategies			
	December		January	
	Losers	Winners	Losers	Winners
Strategy	Short	Long	Long	Long
Mean (no fees)	(0.0011)	0.0359	0.0403	0.0291
Mean (with fees)	(0.0148)	0.0309	0.0267	0.0239
Sharpe (no fees)	0.10	0.52	0.42	0.35
Sharpe (with fees)	0.25	0.44	0.27	0.28
Appraisal (no fees)	(0.58)	0.46	0.34	0.13

Panel B: Small firms

	Equally weighted strategies			
	December		January	
	Losers	Winners	Losers	Winners
Strategy	Short	Long	Long	Long
Mean (no fees)	0.0186	0.0240	0.1718	0.0714
Mean (with fees)	(0.0957)	(0.0076)	0.0467	0.0384
Sharpe (no fees)	0.28	0.35	1.14	0.81
Sharpe (with fees)	0.85	(0.21)	0.29	0.42
Appraisal (no fees)	(0.47)	0.16	1.22	0.86

	Value-weighted strategies			
	December		January	
	Losers	Winners	Losers	Winners
Strategy	Short	Long	Long	Long
Mean (no fees)	0.0193	0.0315	0.1195	0.0540
Mean (with fees)	(0.0427)	0.0159	0.0569	0.0385
Sharpe (no fees)	0.31	0.46	0.90	0.63
Sharpe (with fees)	0.45	0.19	0.45	0.45
Appraisal (no fees)	(0.50)	0.36	0.95	0.63

Panel C: Large firms

	Equally weighted strategies			
	December		January	
	Losers	Winners	Losers	Winners
Strategy	Long	Long	Long	Long
Mean (no fees)	0.0131	0.0234	0.0304	0.0197
Mean (with fees)	0.0067	0.0203	0.0238	0.0166
Sharpe (no fees)	0.17	0.37	0.32	0.23
Sharpe (with fees)	0.03	0.31	0.24	0.19
Appraisal (no fees)	(0.15)	0.22	0.17	(0.12)

	Value-weighted strategies			
	December		January	
	Losers	Winners	Losers	Winners
Strategy	Long	Long	Long	Long
Mean (no fees)	0.0085	0.0216	0.0248	0.0180
Mean (with fees)	0.0035	0.0190	0.0198	0.0154
Sharpe (no fees)	0.08	0.33	0.26	0.20
Sharpe (with fees)	(0.03)	0.28	0.20	0.16
Appraisal (no fees)	(0.25)	0.14	0.03	(0.14)

Table 6
Investment feasibility of momentum trading strategies—all firms

We calculate the maximum dollar amount (millions, adjusted to December 31, 1999) to be invested in different trading strategies so that their performance measures (mean returns and Sharpe ratios) would break even with those of different benchmarks. The figures stem from the price impacts of the strategies. The strategies investigated represent a one-month holding period. The definitions of losers and winners are as follows. First, all NYSE, AMEX, and NASDAQ stocks are sorted at the beginning of every month according to their past 12-month performance (excluding the last month). The bottom decile is denoted as losers, while the top decile is denoted as winners. The different trading strategies investigated consist of losers and winners during December and January. Since losers earn less than the risk-free rate during December, the chosen strategy is a short position. All other strategies are long positions. Equally weighted and value-weighted strategies are both analyzed, and all strategies are analyzed with and without the existence of trading fees. The upper bound is defined as the maximum dollar amount that may be invested in order to achieve a positive Sharpe ratio. No values are shown for investment strategies that cannot achieve the performance measures of the benchmarks. The analysis employs monthly return data (December and January only) for the period December 1963 to December 1999.

Panel A: Equally weighted strategies

	December		January	
	Losers	Winners	Losers	Winners
Strategy	Short	Long	Long	Long
Mean returns				
Market VW	-	-	92	496
Market EW	-	-	-	-
S&P 500	-	-	232	1,102
Small Cap Index	-	-	-	-
Large Cap Index	-	-	150	748
Mean returns (no fees)				
Market VW	-	717	1,117	1,750
Market EW	19	1,037	604	-
S&P 500	-	853	1,256	2,357
Small Cap Index	-	886	538	-
Large Cap Index	-	764	1,174	2,002
Sharpe ratios				
Market VW	-	-	-	200
Market EW	-	-	-	-
S&P 500	-	-	-	468
Small Cap Index	-	-	-	-
Large Cap Index	-	-	5	658
Upper bound	-	352	317	1,474
Sharpe ratios (no fees)				
Market VW	-	300	850	1,410
Market EW	103	886	169	-
S&P 500	-	486	951	1,687
Small Cap Index	103	886	287	-
Large Cap Index	-	539	1,021	1,883
Upper bound	253	1,399	1,342	2,728

Panel B: Value-weighted strategies

	December		January	
	Losers	Winners	Losers	Winners
Strategy	Short	Long	Long	Long
Mean returns				
Market VW	-	6,326	687	82
Market EW	-	10,106	-	-
S&P 500	-	7,938	3,490	8,122
Small Cap Index	-	8,327	-	-
Large Cap Index	-	6,882	1,851	3,421
Mean returns (no fees)				
Market VW	-	9,095	3,918	3,629
Market EW	-	12,874	-	-
S&P 500	-	10,706	6,721	11,669
Small Cap Index	-	11,096	-	-
Large Cap Index	-	9,650	5,082	6,968
Sharpe ratios				
Market VW	-	1,420	-	-
Market EW	-	8,588	-	-
S&P 500	-	3,886	-	-
Small Cap Index	-	8,588	-	-
Large Cap Index	-	4,544	771	2,260
Upper bound	-	14,383	5,206	13,043
Sharpe ratios (no fees)				
Market VW	-	4,530	1,230	-
Market EW	-	11,189	-	-
S&P 500	-	6,715	2,797	3,230
Small Cap Index	-	11,189	-	-
Large Cap Index	-	7,317	3,887	5,709
Upper bound	952	17,152	8,437	16,590

Table 7

Investment feasibility of momentum trading strategies—small firms

We calculate the maximum dollar amount (millions, adjusted to December 31, 1999) to be invested in different trading strategies so that their performance measures (mean returns and Sharpe ratios) would break even with those of different benchmarks. The figures stem from the price impacts of the strategies. The strategies investigated represent a one-month holding period. The definitions of small losers and winners are as follows. First, all NYSE, AMEX, and NASDAQ stocks are sorted according to their market capitalization at the beginning of February, using NYSE breakpoints. The bottom quintile is defined as small firms. The stocks in the latter quintile are then sorted according to their cumulative returns during February-October. The bottom performance decile is denoted as losers, while the top decile is denoted as winners. The different trading strategies investigated consist of small losers and small winners during December and January. Since small losers earn less than the risk-free rate during December, the chosen strategy is a short position. All other strategies are long positions. Equally weighted and value-weighted strategies are both analyzed, and all strategies are analyzed with and without the existence of trading fees. The upper bound is defined as the maximum dollar amount that may be invested in order to achieve a positive Sharpe ratio. No values are shown for investment strategies that cannot achieve the performance measures of the benchmarks. The analysis employs monthly return data (December and January only) for the period December 1963 to December 1999.

Panel A: Equally weighted strategies

	December		January	
	Losers	Winners	Losers	Winners
Strategy	Short	Long	Long	Long
Mean returns				
Market VW	-	-	33	99
Market EW	-	-	-	-
S&P 500	-	-	51	179
Small Cap Index	-	-	-	-
Mean returns (no fees)				
Market VW	-	35	216	322
Market EW	15	88	153	29
S&P 500	5	58	233	402
Small Cap Index	7	63	144	-
Sharpe ratios				
Market VW	-	-	-	32
Market EW	-	-	-	-
S&P 500	-	-	-	73
Small Cap Index	-	-	-	-
Upper bound	-	-	61	227
Sharpe ratios (no fees)				
Market VW	-	-	174	242
Market EW	22	70	77	-
S&P 500	-	8	188	282
Small Cap Index	22	70	94	5
Upper bound	59	148	243	450

Panel B: Value-weighted strategies

	December		January	
	Losers	Winners	Losers	Winners
Strategy	Short	Long	Long	Long
Mean returns				
Market VW	-	-	157	356
Market EW	-	82	-	-
S&P 500	-	-	213	642
Small Cap Index	-	-	-	-
Mean returns (no fees)				
Market VW	-	314	454	731
Market EW	48	491	248	-
S&P 500	20	389	510	1,017
Small Cap Index	25	408	222	-
Sharpe ratios				
Market VW	-	-	57	171
Market EW	-	10	-	-
S&P 500	-	-	100	314
Small Cap Index	-	10	-	-
Upper bound	-	282	247	817
Sharpe ratios (no fees)				
Market VW	-	94	364	538
Market EW	72	419	21	-
S&P 500	-	197	404	682
Small Cap Index	72	419	87	-
Upper bound	179	691	544	1,192

Table 8

Investment feasibility of momentum trading strategies—large firms

We calculate the maximum dollar amount (millions, adjusted to December 31, 1999) to be invested in different trading strategies so that their performance measures (mean returns and Sharpe ratios) would break even with those of different benchmarks. The figures stem from the price impacts of the strategies. The strategies investigated represent a one-month holding period. The definitions of large losers and winners are as follows. First, all NYSE, AMEX, and NASDAQ stocks are sorted according to their market capitalization at the beginning of February, using NYSE breakpoints. The top quintile is defined as large firms. The stocks in the latter quintile are then sorted according to their cumulative returns during February-October. The bottom performance decile is denoted as losers, while the top decile is denoted as winners. The different trading strategies investigated consist of large losers and large winners during December and January. Since all strategies earn more than the risk-free rate during these two months, the chosen strategies are long positions. Equally weighted and value-weighted strategies are both analyzed, and all strategies are analyzed with and without the existence of trading fees. The upper bound is defined as the maximum dollar amount that may be invested in order to achieve a positive Sharpe ratio. No values are shown for investment strategies that cannot achieve the performance measures of the benchmarks. The analysis employs monthly return data (December and January only) for the period December 1963 to December 1999.

Panel A: Equally weighted strategies

	December		January	
	Losers	Winners	Losers	Winners
Strategy	Long	Long	Long	Long
Mean returns				
Market VW	-	117	-	-
Market EW	-	1,146	-	-
S&P 500	-	556	1,617	720
Large Cap Index	-	268	669	-
Mean returns (no fees)				
Market VW	-	588	905	-
Market EW	65	1,617	-	-
S&P 500	-	1,027	2,526	1,211
Large Cap Index	-	739	1,578	122
Sharpe ratios				
Market VW	-	-	-	-
Market EW	-	829	-	-
S&P 500	-	-	-	-
Large Cap Index	-	-	78	-
Upper Bound	244	2,307	2,610	1,859
Sharpe ratios (no fees)				
Market VW	-	-	-	-
Market EW	-	1,206	-	-
S&P 500	-	240	360	-
Large Cap Index	-	366	950	56
Upper Bound	1,219	2,778	3,519	2,350

Panel B: Value-weighted strategies

	December		January	
	Losers	Winners	Losers	Winners
Strategy	Long	Long	Long	Long
Mean returns				
Market VW	-	-	-	-
Market EW	-	860	-	-
S&P 500	-	326	2,382	1,958
Large Cap Index	-	65	272	-
Mean returns (no fees)				
Market VW	-	287	301	-
Market EW	-	1,219	-	-
S&P 500	-	684	3,910	3,448
Large Cap Index	-	424	1,799	-
Sharpe ratios				
Market VW	-	-	-	-
Market EW	-	578	-	-
S&P 500	-	-	-	-
Large Cap Index	-	-	-	-
Upper Bound	-	1,911	4,591	6,099
Sharpe ratios (no fees)				
Market VW	-	-	-	-
Market EW	-	834	-	-
S&P 500	-	27	-	-
Large Cap Index	-	146	785	-
Upper Bound	1,080	2,269	6,119	7,589

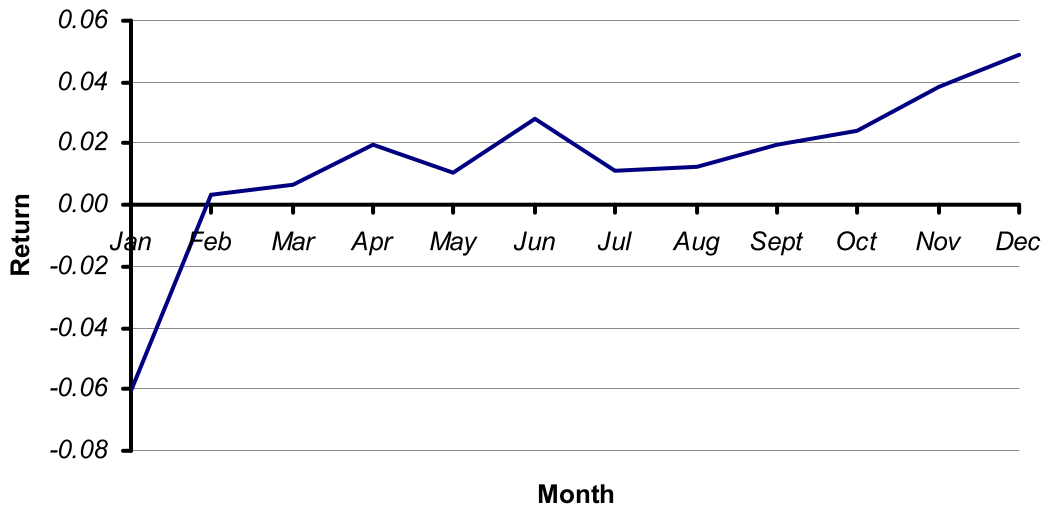


Fig. 1. Monthly returns to winners minus losers

We first sort all NYSE, AMEX, and NASDAQ stocks at the beginning of every month by their past 12-month return (excluding the last month). The bottom decile is denoted losers and the top decile winners. The monthly average of equally weighted returns to winners minus losers are reported above. The analysis uses monthly return data for the period December 1963 to December 1999.

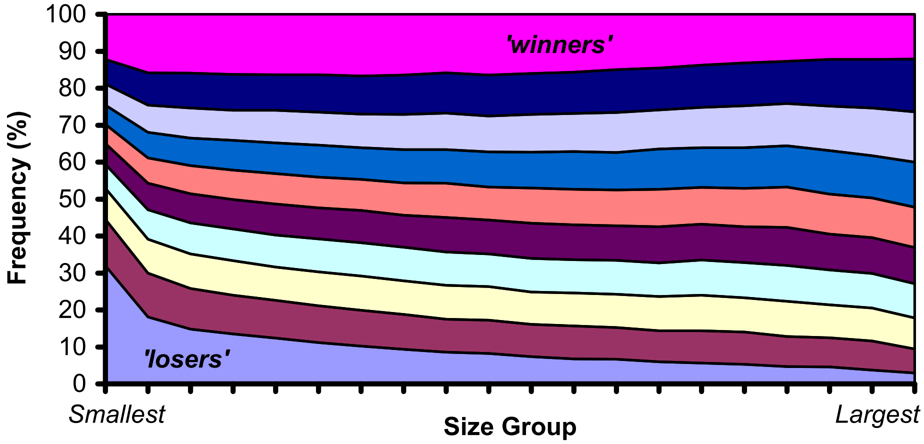
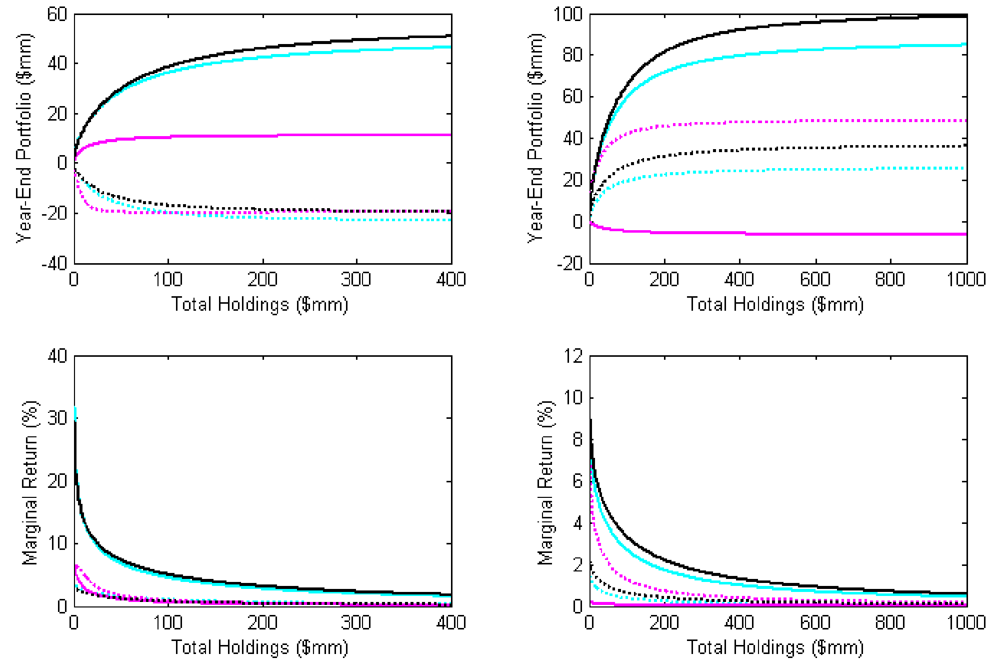


Fig. 2. Momentum frequencies among size groups

We first sort all NYSE, AMEX, and NASDAQ stocks at the beginning of every month by their past 12-month return (excluding the last month). The bottom decile is denoted losers and the top decile winners. Independent of momentum, all stocks are sorted every month into 20 groups according to their market capitalization at the beginning of the month. Every month the relative proportions of the 10 momentum deciles in each size group are calculated. The frequencies plotted below are the time-series averages of these proportions. The analysis uses monthly return data for the period December 1963 to December 1999. All sorts by past performance and market capitalization employ NYSE breakpoints.

Losers Winners

Panel A: Equally weighted strategies



Panel B: Value-weighted strategies

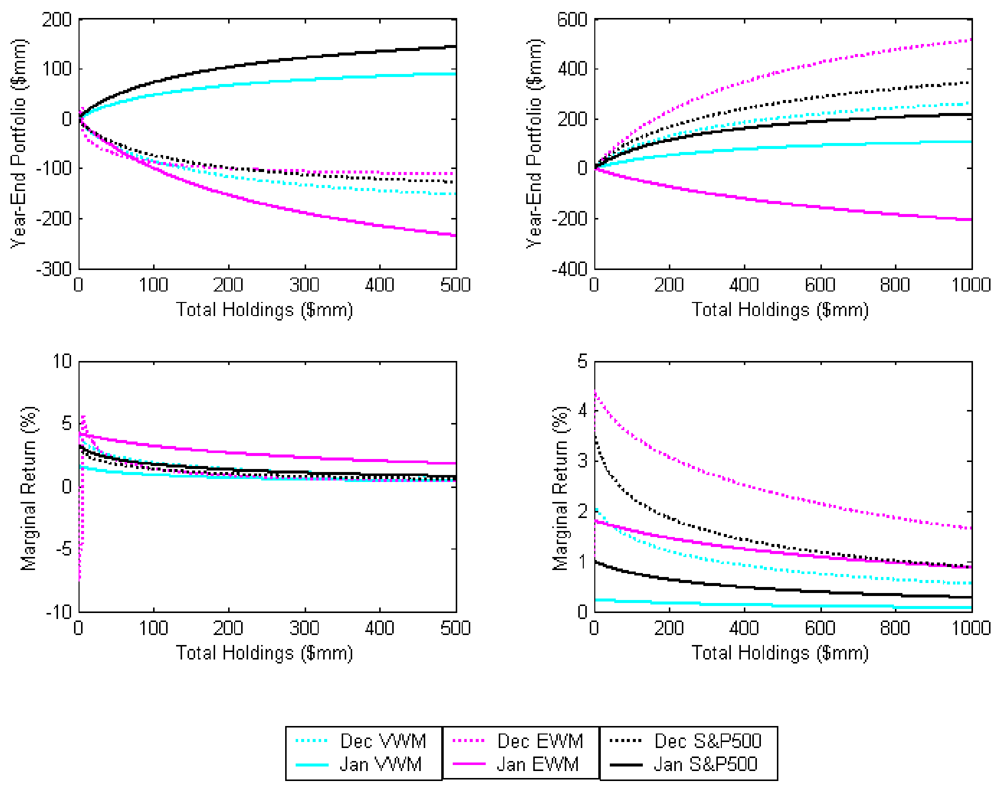


Fig. 3. Optimal allocation of year-end portfolios (all firms)

We calculate the optimal dollar amount (millions, adjusted to December 31, 1999) to be allocated to different momentum-based trading strategies as part of any total investment amount. By assumption, the rest of the funds are invested in a benchmark index. The optimality stems from maximization of the Sharpe ratio of the total investment portfolio. After calculating the optimal allocations, we also report the additional return to the total portfolio from investing in each year-end strategy. The strategies investigated represent a one-month holding period. The definitions of losers and winners are as follows. First, all NYSE, AMEX, and NASDAQ stocks are sorted at the beginning of every month according to their past 12-month performance (excluding the last month). The bottom decile is denoted as losers, while the top decile is denoted as winners. The different trading strategies investigated consist of losers and winners during December and January. Equally weighted and value-weighted strategies are both analyzed. Negative investment figures of the momentum-based strategies indicate a short position. All strategies are analyzed in the presence of price impacts alone without including any trading fees. The analysis employs monthly return data (December and January only) for the period December 1963 to December 1999.

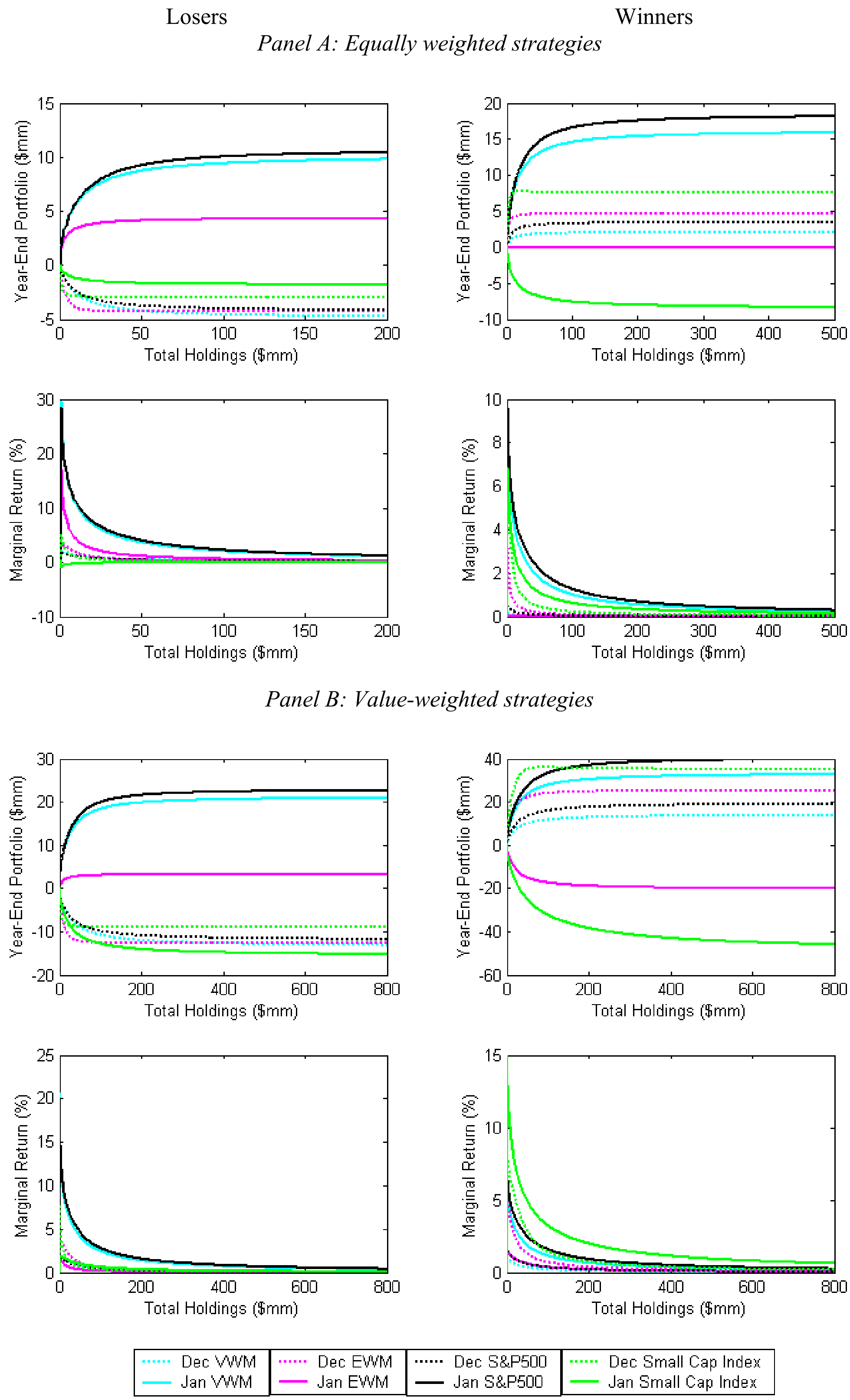


Fig. 4. Optimal allocation of year-end portfolios (small firms)

We calculate the optimal dollar amount (millions, adjusted to December 31, 1999) to be allocated to different momentum-based trading strategies of small firms as part of any total investment amount. By assumption, the rest of the funds are invested in a benchmark index. The optimality stems from maximization of the Sharpe ratio of the total investment portfolio. After calculating the optimal allocations, we also report the additional return to the total portfolio from investing in each year-end strategy. The strategies investigated represent a one-month holding period. The definitions of small losers and winners are as follows. First, all NYSE, AMEX, and NASDAQ stocks are sorted according to their market capitalization at the beginning of February using NYSE breakpoints. The bottom quintile is defined as small firms. The stocks in the latter quintile are then sorted according to their cumulative returns during February-October. The bottom performance decile is denoted as losers, while the top decile is denoted as winners. The different trading strategies investigated consist of small losers and small winners during December and January. Negative investment figures of the momentum-based strategies indicate a short position. All strategies are analyzed in the presence of price impacts alone without including any trading fees. The analysis employs monthly return data (December and January only) for the period December 1963 to December 1999.

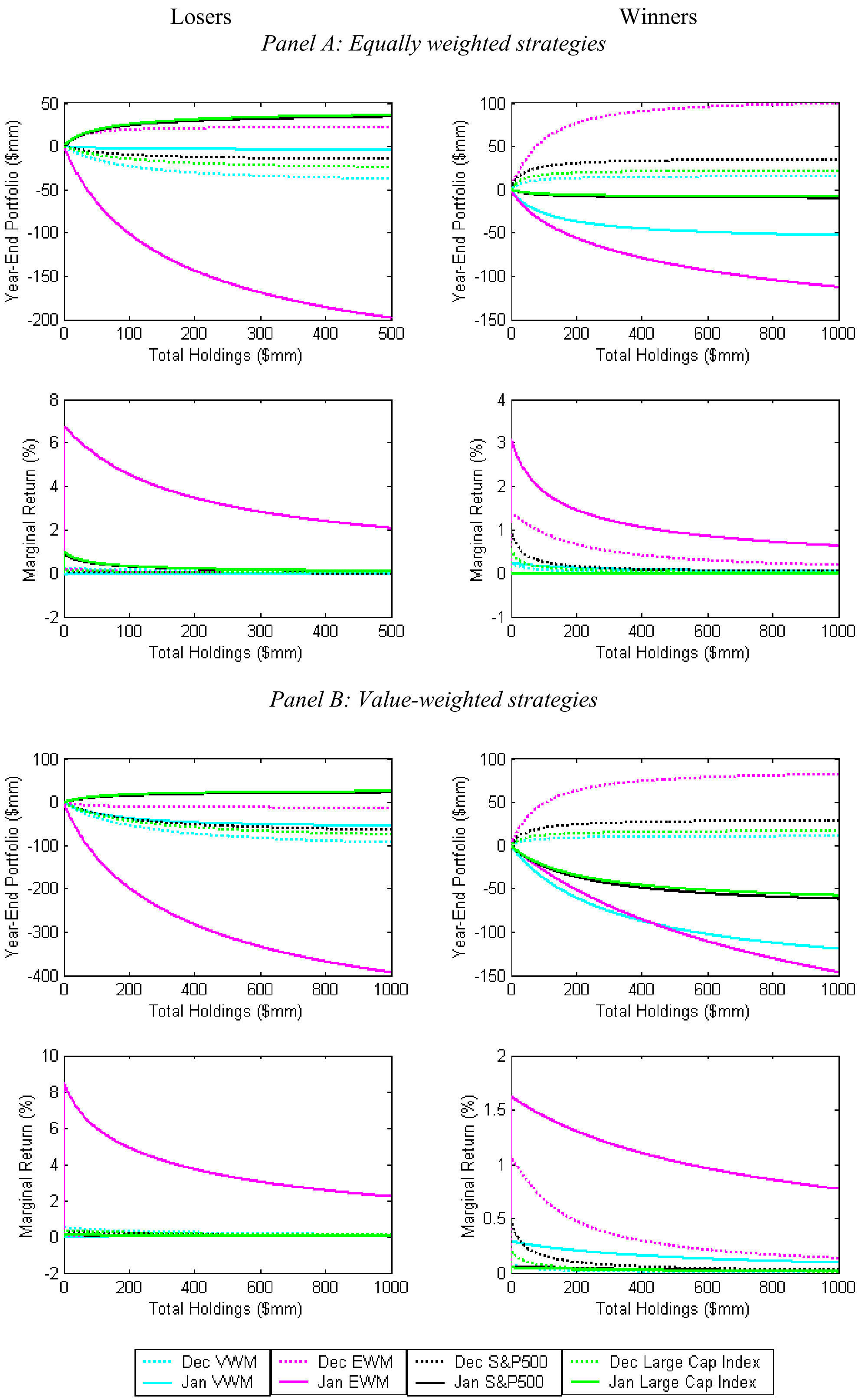


Fig. 5. Optimal allocation of year-end portfolios (large firms)

We calculate the optimal dollar amount (millions, adjusted to December 31, 1999) to be allocated to different momentum-based trading strategies of large firms as part of any total investment amount. By assumption, the rest of the funds are invested in a benchmark index. The optimality stems from maximization of the Sharpe ratio of the total investment portfolio. After calculating the optimal allocations, we also report the additional return to the total portfolio from investing in each year-end strategy. The strategies investigated represent a one-month holding period. The definitions of large losers and winners are as follows. First, all NYSE, AMEX, and NASDAQ stocks are sorted according to their market capitalization at the beginning of February using NYSE breakpoints. The top quintile is defined as large firms. The stocks in the latter quintile are then sorted according to their cumulative returns during February-October. The bottom performance decile is denoted as losers, while the top decile is denoted as winners. The different trading strategies investigated consist of large losers and large winners during December and January. Negative investment figures of the momentum-based strategies indicate a short position. All strategies are analyzed in the presence of price impacts alone without including any trading fees. The analysis employs monthly return data (December and January only) for the period December 1963 to December 1999.